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NONLINEAR POSITION SERVO DESIGN USING THE QLQG/LTR METHOD

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A new nonlinear controller design method called the QLQG/LTR method is proposed. This method is related to the use of statistical linearization, LQG/LTR, and IRIDF methods. It is applied to a position servo with Coulomb friction. The computer simulation results show that the QLQG/LTR control system can adapt automatically to changes in input magnitude.

Key Words: Statistical Linearization, LQG/LTR, QLQG/LTR, Position Servo Design

1. INTRODUCTION

Most practical controllers for servo systems have been designed through application of linear control design methods. However, in general real systems are nonlinear and there always exist modeling errors in mathematical models of real systems. Therefore, sometimes we cannot obtain satisfactory performance in linear control systems neglected modeling errors and/or nonlinear effects.

The QLQG/LTR(Quasi-Linear Quadratic Gaussian control with Loop Transfer Recovery) design method is proposed for considering these problems in nonlinear systems. This design method is the integration of statistical linearization of nonlinear systems, (Gelb, 1968; Atherton, 1975) TFL(Target Filter Loop) design (Athans, 1986) LTR(Loop Transfer Recovery) (Athans, 1986) using the cheap control QLQR(Quasi-Linear Quadratic Regulator) problem (Beaman, 1984) and IRIDF(Inverse Random Input Describing Function) methods. (Suzuki, 1985) This method is suitable to solve the stability-robustness problem in nonlinear systems, especially in nonlinear systems with hard nonlinearities such as Coulomb friction, backlash and saturation.

In this paper, the QLQG/LTR method is applied to a position servo with Coulomb friction. In order to show the effectiveness of the QLQG/LTR method, the linear control system using the LQG/LTR method (Doyle and Stein, 1981) and the nonlinear control system using the QLQG/LTR method are compared. It is found that the QLQG/LTR control system is relatively insensitive to reference input magnitude.

2. QLQG/LTR CONTROL METHOD

Nonlinear plant dynamics can be expressed as follows:

$$\dot{x}(t)=f(x(t))+Bu(t)+\Gamma w(t) \tag{1}$$

where

$x(t)$ is the $(n\times 1)$ plant state vector,
$f(x(t))$ is an $(n\times 1)$ vector,
$u(t)$ is the $(m\times 1)$ control input vector,
$w(t)$ is the $(p\times 1)$ disturbance input vector.

We assume that all the nonlinearities are symmetric and single-valued. Then, the nonlinear plant dynamics(1) can be linearized via statistical linearization techniques.

$$\dot{x}(t)=N(\sigma_x)x(t)+Bu(t)+\Gamma w(t) \tag{2}$$

where

$N(\sigma_x)$ is the $(n\times n)$ statistically linearized plant matrix,
σ_x is the standard deviation of the plant states.

And measurement equations can be expressed as follows:

$$y(t)=Cx(t)+v(t) \tag{3}$$

where

$y(t)$ is the $(m\times 1)$ measured output vector,
$v(t)$ is the $(m\times 1)$ measurement noise vector.

If the statistically linearized system is stabilizable and detectable, then the MBC(Model Based Compensator) can be designed by the QLQG/LTR method. Fig. 1 shows the statistically linearized MBC and design plant.

And the statistically linearized compensated plant dynamics can be expressed as follows:

$$\begin{Bmatrix}\dot{x}(t)\\ \dot{z}(t)\end{Bmatrix}=\begin{bmatrix}N(\sigma_x) & -BG\\ HC & N(\sigma_z)-HC-BG\end{bmatrix}\begin{Bmatrix}x(t)\\ z(t)\end{Bmatrix}+\begin{bmatrix}0 & \Gamma & 0\\ -H & 0 & H\end{bmatrix}\begin{Bmatrix}r(t)\\ w(t)\\ v(t)\end{Bmatrix} \tag{4}$$

where

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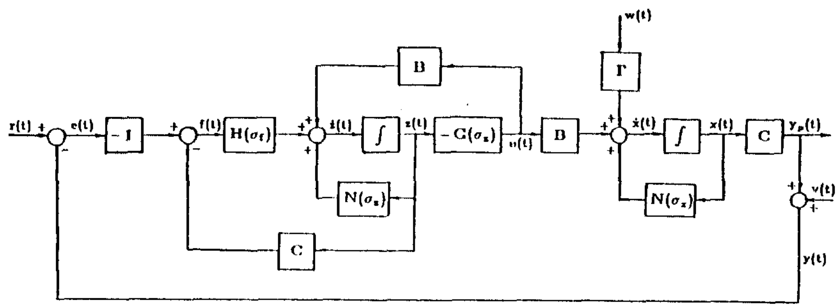

Fig. 1 Statistically linearized compensator and design plant

$z(t)$ is the $(n\times 1)$ compensator state vector,
$r(t)$ is the $(m\times 1)$ command input vector,
$N(\sigma_z)$ is the $(n\times n)$ statistically linearized compensator matirx,
σ_z is the standard deviation of the compensator states,
G is the $(m\times n)$ control gain matrix,
H is the $(n\times m)$ filter gain matrix.

It is convenient to use the separation property (Kwakernaak, 1972) which is one of the special properties of the MBC in order to select desired design matrices (G and H) systematically. We can choose the state vector for closed-loop system $x_c(t)\in R^{2n}$ in order to describe the separation property.

$$x_c(t)=\begin{Bmatrix} x(t) \\ \tilde{x}(t) \end{Bmatrix}$$

where

$$\tilde{x}(t)=x(t)-z(t) \tag{5}$$

Then the statistically linearized QLQG/LTR control system can be expressed as follows:

$$\dot{x}_c(t)=\begin{bmatrix} N-BG & -BG \\ 0 & N-HC \end{bmatrix} x_c(t) + \begin{bmatrix} 0 & \Gamma & 0 \\ H & \Gamma & -H \end{bmatrix} \begin{Bmatrix} r(t) \\ w(t) \\ v(t) \end{Bmatrix} \tag{6}$$
$$y(t)=[C\ 0]x_c(t)$$

The 2n statistically linearized eigenvalues can be separated into two distinct groups ($\det(\lambda I-N+BG)$and $\det(\lambda I-N+HC)$). It is now clear that the compensator design decomposes into finding G and H. We can select H from the TFL design and G from the LTR procedure separately.

The design procedure of the QLQG/LTR control system is as follows:

(1) Determine a mathematical model for the nonlinear plant to be controlled.
(2) Analyze the linearized system via statistical linearization techniques.
(3) Determine the design specifications.
(4) Determine the several zero mean white noise inputs which should represent an operating range of interest.
(5) Select an operating point to design a linear controller.
(6) Estimate the DF(Describing Function) gains for the nonlinearities at the selected operating point.
(7) Do loop shaping of the TFL.
(8) LTR using the cheap control QLQR problem.
(9) Solve the Lyapunov equation for the compensated plant.
(10) Calculate the DF gains for nonlinearties.
(11) Compare the estimated DF gains with the computed ones and repeat steps (6) through (11) until the difference between them is small enough.
(12) Store the gains(filter, control and DF) and the standard deviations(compensator states and filter innovations).
(13) Repeat the design procedure from steps (5) through (12) for each operating point.
(14) Determine the relationships between the gains (filter, control and DF) and the stationary statistics of the system, i. e., $H(\sigma_f)$, $G(\sigma_z)$ and $N(\sigma_z)$ where σ_f and σ_z are the standard deviations of the filter innovations and compensator states, respectively.
(15) Synthesize the desired nonlinear functions via the IRIDF techniques.
(16) Implement the final nonlinear controller and check the time responses of outputs and controls by computer simulation.

The QLQG/LTR method uses loop shaping techniques (Stein, 1981) to address the performance and stability-robustness problem. By condsidering the fictitious process and measurement noises instead of the real driving noises, we can achieve desirable loop transfer functions which satisfy the performance and stability-robustness requirements. However, since the driving noises and the fictitious noises are in general different, we must solve the Lyapunov equation for the compensated plant in order to calculate the stationary statistics of the system.

Now, the major design procedure of the QLQG/LTR method will be discussed in detail.

2.1 Design of the TFL

The structure of the TFL is shown in Fig. 2.

If we break the loop at the output or, equivalently, at the error signal, we readily obtain the loop TFM(Transfer Function Matrix) $G_f(s)$.

$$G_f(s)=C(sI-N)^{-1}H \tag{7}$$

For loop shaping of $G_f(s)$, fictitious process and measurement noises are considered and the KFDE(Kalman Filter Frequency Domain Equality) (Stein, 1984) is used. Then, the statistically linearized design plant dynamics with fictitious noises are:

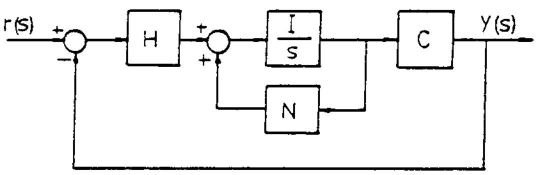

Fig. 2 Structure of the target filter loop

$$\dot{x}(t)=N(\sigma_x)x(t)+Bu(t)+L\xi(t) \qquad (8)$$
$$y(t)=Cx(t)+\theta(t)$$

where $\xi(t)$ is the fictitious process white noise, and $\theta(t)$ is the fictitious measurement white noise, i.e.,

$$E[\xi(t)]=0,\ E[\xi(t)\xi^T(\tau)]=I\delta(t-\tau)$$
$$E[\theta(t)]=0,\ E[\theta(t)\theta^T(\tau)]=\mu I\delta(t-\tau)$$

The matrix L and scalar μ are available as design parameters for the TFL design. The filter gain matrix based on the fictitious noises is :

$$H=\frac{1}{\mu}PC^T \qquad (9)$$

where P is the solution of the FARE(Filter Algebraic Riccati Equation) :

$$NP+PN^T+LL^T-\frac{1}{\mu}PC^TCP=0 \qquad (10)$$

By using the KFDE, the following result is obtained. (Athans, 1986)

$$G_f(s)\simeq\frac{1}{\sqrt{\mu}}C(sI-N)^{-1}L \qquad (11)$$

The above approximation provides valuable insight into the selection of the design parameters L and μ.

2.2 LTR Using the Cheap Control QLQR Problem

The QLQR problem combines use of statistical linearization and LQR(Linear Quadratic Regulator) optimal control theory. From Beaman's results, (Beaman, 1984) the QLQR problem is summerized as follows :

Cost $\quad : J=\frac{1}{2}E[x^TQx+\rho u^Tu] \qquad (12)$

where

Q is a state weighting matrix,
ρ is a control weighting parameter.

State $\quad : \dot{x}=Nx+Bu+\Gamma w \qquad (13)$
Control $\quad : u=-Gx \qquad (14)$
Control gain $\quad : G=\frac{1}{\rho}B^TS \qquad (15)$

where

S is the Riccati matrix.

Modified CARE(Control Algebraic Riccati Equation) :

$$Q+SN+N^TS-\frac{1}{\rho}SBB^TS+\Psi(S,N,X)=0 \qquad (16)$$

where

X is the state covariance matrix,

$$\Psi_{ij}(S,N,X)=2tr\left(S\frac{\partial N}{\partial X_{ij}}X\right) \qquad (17)$$

Lyapunov equation :

$$(N-BG)X+X(N-BG)^T+\Gamma W\Gamma^T=0 \qquad (18)$$

where

W is the disturbance covariance matrix.

The LTR is accomplished by solving the cheap control QLQR problem to recover the target filter loop shape. For the solution of this problem to be valid, $[N, B]$ must be stabilizable and $[N, C]$ must be detectable.

For the LTR, we examine the limiting behavior of the modified CARE (16) with $Q=C^TC$ as $\rho\to0$.

$$C^TC+SN+N^TS-\frac{1}{\rho}SBB^TS$$
$$+\Psi(S,N,X)=0 \qquad (19)$$

By the examination of the order of magnitude of each term in Eq.(19) as $\rho\to0$, the LTR condition for the statistically linearized system is basically the same as in the linear case, i.e., (Kim, 1987)

$$C^TC-\left(\frac{1}{\sqrt{\rho}}SB\right)\left(\frac{1}{\sqrt{\rho}}B^TS\right)\to0 \qquad (20)$$

Substituting Eq. (15) into Eq. (20),

$$(\sqrt{\rho}G)^T(\sqrt{\rho}G)\to C^TC \qquad (21)$$

which implies that

$$\lim_{\rho\to0}\sqrt{\rho}G\to UC \qquad (22)$$

where U is the $(m\times m)$ unitary matrix, i.e., $U^TU=I$.

Now we consider TFM of the MBC, $K(s)$.

$$K(s)=G(sI-N+BG+HC)^{-1}H \qquad (23)$$

If $Re\ \lambda_i[N-BG]<0$, $Re\ \lambda_i[N-HC]<0$ and $\lim_{\rho\to0}\sqrt{\rho}G\to UC$, then the limiting behavior of the $K(s)$ as $\rho\to0$ is as follows : (Doyle, 1981; Kim, 1988)

$$\lim_{\rho\to0}K(s)\to[C(sI-N)^{-1}B]^{-1}[C(sI-N)]^{-1}H]$$
$$=G^{-1}(s)G_f(s) \qquad (24)$$

Using the limiting relation(24), the limiting behavior of the loop TFM at the plant output, $T(s)$ is :

$$\lim_{\rho\to0}T(s)\to G(s)G^{-1}(s)G_f(s)=G_f(s) \qquad (25)$$

2.3 The Lyapunov Equation for the Compensated Plant

Determining the stationary statistics of the total closed-loop system requires a little more thought than in the general LQG(Linear Quadratic Gaussian control) design situation. This is due to the fact that the fictitious (design) noise and the real driving noise are not the same, in general, The target Kalman filter is not an optimal filter for the real driving noise. The TFL is designed to make the desired loop shape with fictitious noise.

Now, let us derive the correlation between the estimator (compensator) state $z(t)$, and the estimation error state $\tilde{x}(t) = x(t) - z(t)$. For convenience we consider the statistically linearized compensated plant dynamics which are expressed in Eqn.(4). By redefining the state variable using the estimation error state $\tilde{x}(t)$, we have the following :

$$\begin{Bmatrix} \dot{\tilde{x}}(t) \\ \dot{z}(t) \end{Bmatrix} = \begin{bmatrix} N(\sigma_x) - HC & 0 \\ HC & N(\sigma_z) - BG \end{bmatrix} \begin{Bmatrix} \tilde{x}(t) \\ z(t) \end{Bmatrix} + \begin{bmatrix} H & \Gamma & -H \\ -H & 0 & H \end{bmatrix} \begin{Bmatrix} r(t) \\ w(t) \\ v(t) \end{Bmatrix} \tag{26}$$

We can now directly write the differential equations for the state covariance matrices as shown in reference (Bryson, 1975). The estimation error state covariance propagation equation :

$$\dot{\tilde{X}} = (N(\sigma_x) - HC)\tilde{X} + \tilde{X}(N(\sigma_x) - HC)^T + HRH^T + \Gamma W \Gamma^T + HVH^T \tag{27}$$

with

$$\tilde{X}(0) = X(0)$$

where

$$\tilde{X} = E[\tilde{x}(t)\tilde{x}(t)^T], \quad X(0) = E[x(0)x(0)^T]$$

The estimator state covariance propagation equation :

$$\dot{Z} = (N(\sigma_z) - BG)Z + Z(N(\sigma_z) - BG)^T + HCY + Y(HC)^T + HRH^T + HVH^T \tag{28}$$

with

$$Z(0) = 0$$

where

$$Z(t) = E[z(t)z(t)^T], \quad Y(t) = E[z(t)\tilde{x}(t)^T]$$

The estimator state-estimation error state covariance propagation equation :

$$\dot{Y} = (N(\sigma_x) - HC)Y + Y(N(\sigma_z) - BG)^T + X(HC)^T - HRH^T - HVH^T \tag{29}$$

with

$$Y(0) = 0$$

In the optimal filtering process, H is defined by :

$$H = \tilde{X}C^T V^{-1} \tag{30}$$

And we assume that command inputs are deterministic($R = 0$). Substituting the optimal filter gain matrix(30) and $R = 0$ into Eq. (29), Eq. (29) can be rewritten as :

$$\dot{Y} = (N - HC)Y + Y(N - BG)^T \tag{31}$$

This is a homogeneous differential equation in $Y(t)$ with initial condition $Y(0) = 0$, which, of course, has the solution

$$Y(t) = 0 \text{ for all } t > 0 \tag{32}$$

The result imples that for an LQG design the estimator states and errors are uncorrelated stochastic processes. Substitution of $Y = 0$ and $R = 0$ leads to a simplification of Eq. (28) for the estimator state covariance :

$$\dot{Z} = (N - BG)Z + Z(N - BG)^T + HVH^T \tag{33}$$

Under the assumption of the optimal filtering process and deterministic inputs, we can know $z(t)$ and $\tilde{x}(t)$ are uncorrelated from Eq. (32). This implies that

$$X = Z + \tilde{X} \tag{34}$$

In the case of the QLQG/LTR design, the above simplification cannot be applied. Because the (fictitious) design noise and the real driving noise are mismatched intentionally, and the fictitious white noise intensity of the command input R is not zero.

In fact, the filter gain matrix is obtained from Eq. (9) which is not the same as Eq. (30). Substitution of Eq. (9) into Eq. (29) does not lead to Eq. (31). $\tilde{X}$ is found from Eq. (27) using the real process and measurement noises and fictitious command inputs, while P is found from the FARE(10) using the fictitious (design) noises.

Since the driving noises and the fictitious noises are in general different, P will not be equal to $\tilde{X}$ implying that Y will not necessarily be zero for all time. Thus, implementation of the QLQG/LTR method will in general lead to correlated estimator states and estimation errors. This can readily be seen from the coupling between Eqs. (27), (28) and (29).

The importance, however, is that none of the Eqs. (27) through (29) can be simplified for easy determination of Z. Secondly, $\tilde{X}$ is not available as a result of the TFL design process (only P is). Thirdly, even if it were available and if we could also determine Z easily, Eq. (34) does not hold for correlated $z(t)$ and $\tilde{x}(t)$. Therefore, we must solve the Lyapunov equation for the compensated plant(35) in order to caculate the gains and stationary statistics of the states. The Lyapunov equation for the compensated plant is derived from Eq.(4) :

$$N_t X_t + X_t N_t^T + \Gamma_t W_t \Gamma_t^T = 0 \tag{35}$$

where

$$N_t = \begin{bmatrix} N & -BG \\ HC & N - BG - HC \end{bmatrix}, \quad X_t = \begin{bmatrix} X & Y \\ Y & Z \end{bmatrix},$$

$$\Gamma_t=\begin{bmatrix} 0 & \Gamma & 0 \\ -H & 0 & H \end{bmatrix},\ W_t=\begin{bmatrix} R & 0 & 0 \\ 0 & W & 0 \\ 0 & 0 & V \end{bmatrix}$$

If we consider the correction term(17) in the cheap control QLQR problem, then the modified CARE(16) and Lyapunov equation for the compensated plant(35) must be solved simultaneously with the guessed unknown variables(X_t ; $n(2n+1)$, S ; $n(n+1)/2$) where n is the number of design plant states. It is very difficult to find a solution for high order systems. In addition, it requires a great deal of computation time. If we neglect the correction term in the cheap control QLQR problem, the CARE and Lyapunov equation are not coupled. Then, these two equations can be solved separately.

Fortunately, the correction term Ψ is not dominant for the good LTR case, i.e., $\lim_{\rho\to 0}\Psi \to 0$, (Kim, 1987) Therefore, we can neglect the correction term under this situation. Then, we can calculate the control gains from the LTR procedure and the stationary statistics of the system from the Lyapunov equation separately. By neglecting the correction term, the required computation is much simpler and we can obtain a solution satisfactor.

3. PROBLEM FORMULATION FOR A NONLINEAR POSITION SERVO DESIGN

A position servo disign problem from robotics is selected as a nonlinear control system design example using the QLQG/LTR method. A block diagram of the nonlinear plant is shown in Fig. 3. The state variables and parameters of the plant are given below :

$\theta_L(x_1)$ is the load displacement,
$\dot\theta_L(x_2)$ is the load angular velocity,
$\theta\delta(x_3)$ is the angular displacement movement of the gearbox output,
$\dot\theta_m(x_4)$ is the motor angular velocity,
$T_m(u)$ is the actual mechanical torque delivered by the motor,
J_m is the motor and gearbox input inertia,
$J_m=1.42\times10^{-5}$kg.m²
B_m is the motor plus gearbox input shaft damping coefficient,
$B_m=2\times10^{-4}$N.m/rad/s
N_G is the gear ratio,
$N_G=100$
K_G is the gearbox stiffness measured at the output,
$K_G=6000$N.m/rad
J_L is the load and gearbox output shaft inertia,
$J_L=0.0426$kg.m²
T_c is the magnitude of the load Coulomb friction,
$T_c=0.1$ N.m

The nonlinear plant is linearized via statistical linearization techniques. Then, statistically linearized plant can be expressed as follows :

$$\dot x(t)=N(\sigma_x)x(t)+Bu(t) \qquad (36)$$
$$y(t)=Cx(t)$$

where

$$N=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -N_c/J_L & K_G/J_L & 0 \\ 0 & -1 & 0 & 1/N_G \\ 0 & 0 & -K_G/J_mN_G & -B_m/J_m \end{bmatrix},B=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/J_m \end{bmatrix}$$

and $C=[1\quad 0\quad 0\quad 0]$

N_c is the DF for the Coulomb friction,
$x(t)$ is the (4×1) plant state vector,
$y(t)$ is the output,
$u(t)$ is the control input.

The design specifications considered are as follows :

(1) Steady state tracking error should be zero for an arbitrary constant input.
(2) Gain crossover frequency should be about 10 rad/sec.
(3) The singular value of the sensitivity TF(Transfer Function) should be less than -20db for all $\omega<1$ rad/sec for the good command following and disturbance rejection.
(4) The singular value of the closed-loop TF should be less than -20db for all $\omega>100$ rad/sec for the stability robustness to unmodeled dynamics.

Since the plant has a free integrator, it is not necessary to augment with an integrator in order to meet the design specification(1). Therefore the DPM(Design Plant Model) can be chosen by Eq. (36), and the MBC is expressed as follows :

$$\dot z(t)=N(\sigma_z)z(t)+Bu(t)+H(y(t)-Cz(t)-r(t)) \qquad (37)$$

where

$z(t)$ is the (4×1) compensator state vector,
$N(\sigma_z)$ is the (4×4) statistically linearized compensator matrix,

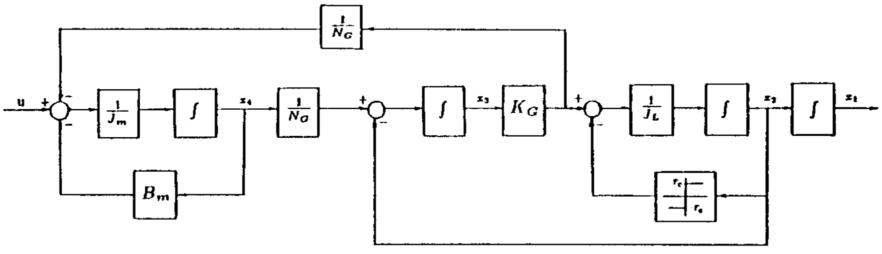

Fig. 3 Block diagram of the nonlinear plant

H is the (4×1) filter gain matrix,
$r(t)$ is the command input.

And control law is as follows:

$$u(t)=-Gz(t) \tag{38}$$

where

G is the (1×4) control gain matrix.

By combining Eqs. (36) and (37), the statistically linearized compensated plant dynamics are expressed as follows:

$$\begin{Bmatrix}\dot{x}(t)\\ \dot{z}(t)\end{Bmatrix}=\begin{bmatrix}N(\sigma_x) & -BG\\ HC & N(\sigma_z)-HC-BG\end{bmatrix}\begin{Bmatrix}x(t)\\ z(t)\end{Bmatrix}+\begin{bmatrix}0\\ -H\end{bmatrix}r(t) \tag{39}$$

4. CONTROL SYSTEM DESIGN

4.1 Linear Controller Design Using the LQG/LTR Method

We should have a linear plant to apply the LQG/LTR method. The Coulomb friction nonlinearity(T_c.sgn(x)) is assumed as a linear one(x). Then we can design the LQG/LTR compensator for the assumed linear plant. In order to apply the LQG/LTR design procedure, the DPM dynamics are expressed as follows:

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t)\\ y(t)&=Cx(t)\end{aligned} \tag{40}$$

where

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -1/J_L & K_G/J_L & 0\\ 0 & -1 & 0 & 1/N_G\\ 0 & 0 & -K_G/J_mN_G & -B_m/J_m\end{bmatrix},\ B=\begin{bmatrix}0\\ 0\\ 0\\ 1/J_m\end{bmatrix}$$

and $C=[1 \quad 0 \quad 0 \quad 0]$

The above DPM is found to be completely controllable from the input $u(t)$ and completely observable through the output $y(t)$, and it is a minimum phase plant. Therefore, we can design the LQG/LTR compensator with a guarantee of the LTR. The structure of the linear feedback control system using the LQG/LTR method can be shown in Fig. 4.

The LQG/LTR method involves two basic steps(TFL design and LTR). In the first step the TFL is designed for the desired loop shaping. In this case the TFL can be designed by cancelling the open-loop poles except for the free integrator, which provides an optimal shape. (Kim, 1988) Then, the design parameter L is selected as follows:

$$L=Z_c^{-1}z_d=\begin{bmatrix}1\\ 0\\ 0\\ 0\end{bmatrix} \tag{41}$$

where

Z_c is the $(n\times n)$ matrix containing as its columns the coefficients of the constituent zero polynomials of the $G_{fol}(s)$ $(=C(sI-A)^{-1}L)$ transfer function,
z_d is the desired zero polynomial.

To determine the filter gain matrix H the desired crossover frequency was specified as 10rad/sec. A value of 0.008 for μ is found to provide a crossover frequency of about 11rad/sec for the TFL which is shown in Fig. 5. This leaves us with some safety margin in the recovery phase.

After selecting L and μ to satisfy the desired target filter loop shaping, we calculate the filter gain matrix H from Eqs. (9) and (10). The resulting filter gain matrix H is:

$$H=\begin{bmatrix}H_1\\ H_2\\ H_3\\ H_4\end{bmatrix}=\begin{bmatrix}11.18\\ 0\\ 0\\ 0\end{bmatrix}$$

Next, the LTR is attempted with the cheap control linear quadratic regulator problem. We usually recover the TFL up to a decade beyond the crossover frequency. This level of recovery is obtained with a value of 0.01 for ρ. Then the control gain matrix G is calculated from Eq. (15) and the standard CARE which is Eq. (16) without correction term[Eq. (17)].

$$\begin{aligned}G&=[G_1\ G_2\ G_3\ G_4]\\ &=[10\ 0.034\ 8.15\ 0.0013]\end{aligned}$$

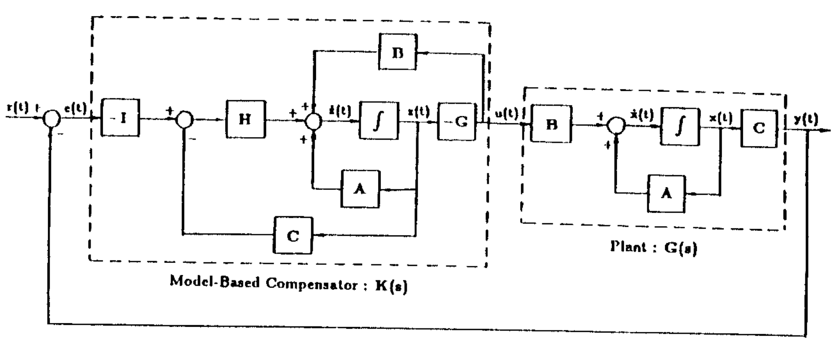

Fig. 4 Linear feedback control system using the LQG/LTR method

The recovered open-loop TF is shown is Fig. 6. The recovery is good up to a decade beyond crossover frequency -the additional roll-off introduced at that point will enhance the stability-robustness.

Now let us check the performance and stability-robustness for the nonlinear plant with the LQG/LTR compensator. For this purpose, we check the frequency responses for 3 different command inputs. We assume the command inputs as zero mean white noises for the statistical linearization of the nonlinear plant. The white noise intensities of the selected command inputs(R) are 10^{-1}, 10^{-4} and 10^{-7} which represent large, medium and small input cases, respectively. The singular value plot for the open-loop TF and the normalized step responses for the nonlinear plant with the LQG/LTR compensator are shown in Fig. 7 and Fig. 8, respectively.

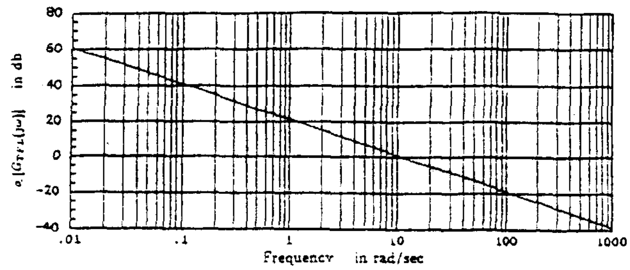

Fig. 5 Singular value of the target filter loop TF

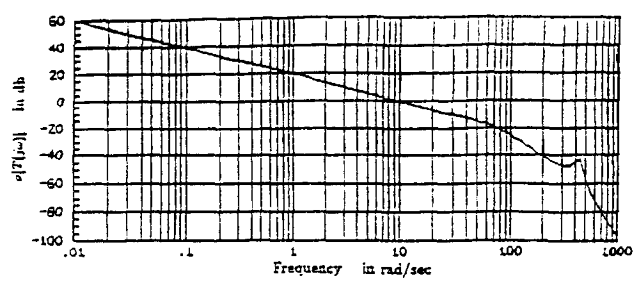

Fig. 6 Singular value of the recovered open-loop TF

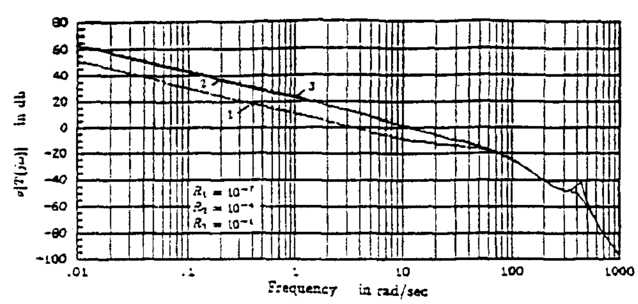

Fig. 7 Singular value of the open-loop TF for the nonlinear plant with the LQG/LTR compensator

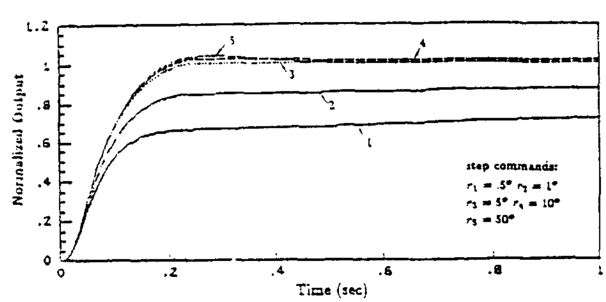

Fig. 8 Step responses for the LQG/LTR control system

The LQG/LTR control system satisfies the stability-robustness condition for any input, but it does not satisfy the performance requirements for small inputs. In the time response, there is a steady state error of about 0.2 for any constant input. This is due to the effect of the Coulomb friction. A linear compensator whithout a free integrator in it creates a steady state error for a constant input, even if a free integrator action is in the plant which has Coulomb friction. In addition, some overshoot exists for large input and the system response is very slow for small input. Therefore, the linear LQT/LTR compensator cannot be used for a large operating range. In order to satisfy the performance and stability-robustness for the entire operating range, a nonlinear compensator considering the effect of the Coulomb friction is required.

4.2 Nonlinear Controller Design Using the QLQG/LTR Method

We should have the statistically linearized plant(36) and select several opearting points to cover an operating range of interest to apply the QLQG/LTR method. The zero mean white noise intensities of the command inputs(R) are selected between 10^{-1} and 10^{-7}. Steps(6) through (11) of the QLQG/LTR design procedure which is shown in secton 2 are executed for a linear design at each selected operating point. Only the final results will be discussed here. Note that for each linear design the design uses an iterative process which terminates when the difference between the estimated and computed DF gains is small enough.

The gains(filter, control and DF) and the stationary statistics(compensator states and filter innovation) are stored for all linear designs. The filter gains have the same values as the LQG/LTR case. The control gains at all different operating points are shown in Table 1.

Since G_1 is constant for any input and G_2, G_3 and G_4 are almost constant, we can select the constant control gain matrix G as follows:

$$G=[G_1\ G_2\ G_3\ G_4]$$
$$=[10\ 0.03\ 9.55\ 0.0015]$$

The relationship between the DF gain for Coulomb friction and the standard deviation of σ_{z2} is shown in Fig. 9, and the desired nonlinear function for the Coulomb friction nonlinearity is obtained via the IRIDF techniques. This procedure can be done in an iterative way, each time improving upon the approximation until the desired level of accuracy has been reached. The final result of the desired nonlinear function is shown in Fig. 10. Then we can synthesize the nonlinear

Table 1 Control gains at all operating points

R	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
G_1	10	10	10	10	10	10	10
G_2	.028	.028	.028	.030	.032	.034	.030
G_3	9.93	9.90	9.81	9.55	8.91	7.71	5.58
G_4	.0015	.0015	.0015	.0015	.0014	.0013	.0011

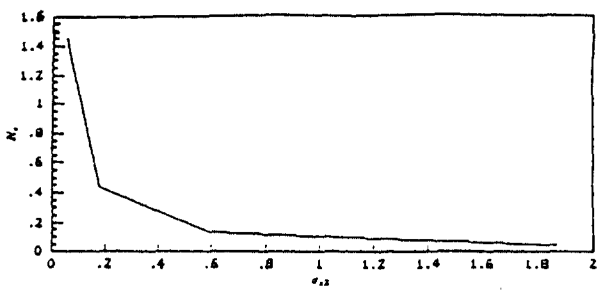

Fig. 9 Quasi-linear gain N_c versus σ_{z2}

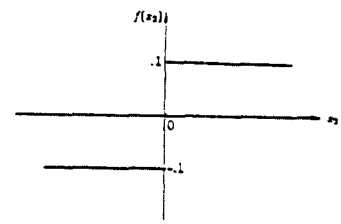

Fig. 10 Desired nonlinear function for the Coulomb friction

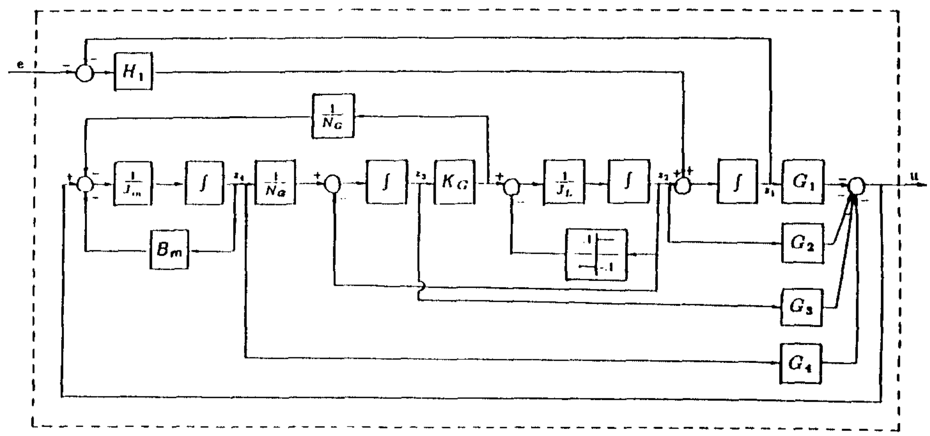

Fig. 11 The nonlinear QLQG/LTR compensator

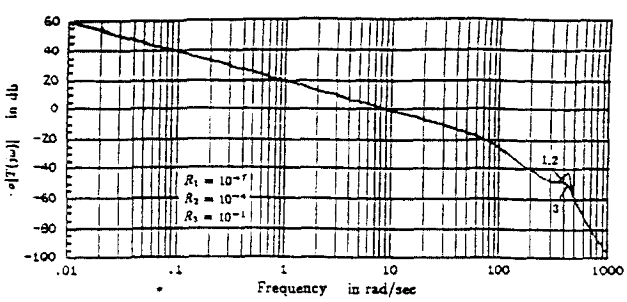

Fig. 12 Singular value of the open-loop TF for the QLQG/LTR control system

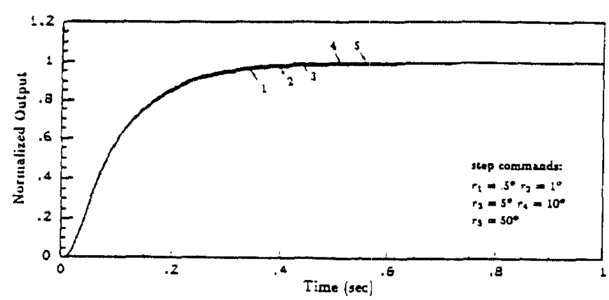

Fig. 13 Step responses for the QLQG/LTR control system

QLQG/LTR compensator as shown in Fig. 11.

The singular value plot for the open-loop TF and the normalized step responses for the nonlinear position servo with the QLQG/LTR compensator are shown in Fig. 12 and Fig. 13, respectively. The system responses are desirable in both the frequency domain and the time domain. The settling time is about 0.4 seconds and no overshoot exists for any input. Since the nolinear QLQG/LTR compensator adapts to changes in input magintude satisfactorily, the system responses are insensitive to the input magnitude. In addition, no steady state error exists for any constant input.

5. SUMMARY AND CONCLUSIONS

The nonlinear QLQG/LTR design method has been developed and the LQG/LTR and QLQG/LTR compensators are designed for a nonlinear position servo with Coulomb friction. In the frequency domain, the LQG/LTR control system satisfies the design specifications for a small input range, but the QLQG/LTR control system satisfies them for the entire operating range ($0.5^\circ < r < 50^\circ$). Also, the time response of the LQG/LTR control system is not good. There exists a steady state error for a constant input even if the system has a free integrator, because the LQG/LTR compensator cannot adapt to the effect of the Coulomb friction. In addition, there is some overshoot for large inputs and the system response is very slow for small inputs. However, for the QLQG/LTR case, the system responses are insensitive to the input magnitude and have no overshoot, no steady state error and fast settling time for all operating ranges. This is so because the QLQG/LTR compensator can adapt to the effect of the

Coulomb friction, since the statistical linearization retains a considerable part of it in the design plant model.

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